New Product Introduction Against a Predator: A Bilevel Mixed-Integer Programming Approach

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Abstract: We consider a scenario with two firms determining which products to develop and introduce to the market. In this problem, there exists a finite set of potential products and market segments. Each market segment has a preference list of products and will buy its most preferred product among those available. The firms play a Stackelberg game in which the leader firm first introduces a set of products, and the follower responds with its own set of products. The leader's goal is to maximize its profit subject to a product introduction budget, assuming that the follower will attempt to minimize the leader's profit using a budget of its own. We formulate this problem as a multistage integer program amenable to decomposition techniques. Using this formulation, we develop three variations of an exact mathematical programming method for solving the multistage problem, along with a family of heuristic procedures for estimating the follower solution. The efficacy of our approaches is demonstrated on randomly generated test instances. This article contributes to the operations research literature a multistage algorithm that directly addresses difficulties posed by degeneracy, and contributes to the product variety literature an exact optimization algorithm for a novel competitive product introduction problem. © 2009 Wiley Periodicals, Inc. Naval Research Logistics 56: 714–729, 2009

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1. INTRODUCTION

Because a firm's portfolio of products is its defining characteristic, there is hardly any other strategic decision that is more important to a firm than deciding what products to offer. Product introduction decisions must account for possible actions by competitors, who will usually respond by introducing products of their own. In many corporate settings, predatory actions are illegal; i.e., a company may not engage in business practices for the primary purpose of putting another company out of business, or for preventing others from entering a market. However, they are still observed nonetheless ([6, 13, 16, 21, 35]), partly because they are in general very hard to prove legally. Hence, in this article, we discuss models and algorithms for optimizing product introduction in the face of predatory competitors. To establish motivation for our study, we first discuss three specific examples of predatory behavior from technology, travel, and restaurant industries.

- The United States government alleged in a court case that spanned 1969–1982 that the product introduction and price setting of IBM's 360/90 supercomputer were predatory actions in response to the introduction of the Model 6600 super-computer by Control Data Corporation [35]. The lawsuit was eventually dropped, conforming to the general wisdom that predation is difficult to provably identify in a court of law. One must consider the possibility that the product introduction decisions of the follower (IBM in this case) were merely suboptimal or based on faulty market research, instead of malicious attempts to act in a predatory manner.
- Legend Airlines was a small start-up that flew 56-seat planes out of Love Field in Dallas for only 8 months of its less than 5-year existence in 1995–2000 [13]. American Airlines, with a home base in Dallas/Fort

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Worth Airport (DFW), fought ferociously to counter the threat that Legend posed to its lucrative business traveler segment. After 4 years of court battle over whether Legend could even use Love Field, as soon as Legend began flying, American introduced its own version of 56-seat flights out of Love Field. American is also known to compete with Southwest Airlines by diverting some American flights from DFW to Love Field [33]. Many believe that American's aggressive tactics—whether truly predatory or not—were a major factor in Legend's quick demise in 2000. Soon after, American discontinued its "new products" expressly designed to compete with Legend.
- Starbucks has recently been accused of predatory practices in its retail location decisions and real estate dealings [21]. In September 2006, the owner of a small coffee shop (in Bellevue, Washington) filed a federal lawsuit alleging that Starbucks has a habit of flooding certain localities with too many stores and signing exclusive leasing deals with high-rise owners to bar other coffee shops from coveted downtown areas with high foot-traffic. Considering retail location as a product attribute, some of Starbucks's product introduction decisions appear to be predatory in the sense that they may be driven by the objective of putting competitors out of business or keeping them out of high-profit market segments.

Although anecdotal, never-ending controversies that usually surround such cases of (most possibly) predatory conduct are an indication that predation is both widespread and notoriously difficult to establish in a court of law. The latter is especially true for predatory product introduction (more so than predatory pricing, for instance) because it is harder to differentiate genuine competition from overly aggressive competition when it comes to innovation and new product development [40]. The issue is so controversial and elusive that Ordover and Willig [34] advance the following argument: "even genuine innovations—new products that in some ways are superior to existing products in the eyes of both engineers and consumers—are in some circumstances anticompetitive." Furthermore, predatory behavior may be quite unintentional. A company may, quite by accident, select a slightly suboptimal set of product introductions (with respect to their own profit goal), with the unintentional result of crippling its competitor. This "accidental" predatory behavior can occur due to a failure of the company to find an exact optimal solution, or due to different perceptions of the market. We will provide a numerical example of this possibility in Section 2.1.

Hence, a leader firm must be concerned about predatory followers in the market whether they assume that a follower is maliciously acting as a predator, or accidentally acting in the leader's worst interests. Note that we are not suggesting that the follower, being predatory, is irrationally ignoring its own interest. In all industry examples of alleged or true predatory behavior, the follower does have a long-term interest in driving the leader out of business, or out of high-profit market segments. Our model captures the follower's short-to-medium-term problem (i.e., the first phase of predation) of accomplishing this aim. Also, in most cases, the long term is immaterial for the leader, because it will cease to exist unless it successfully counters the follower's assault. Our model therefore reflects the leader's perspective of survival. Besides, considering a worst-cast scenario is also directly useful as a tool for a risk-averse decision-maker who believes that predatory decisions can happen by accident, or who simply wishes to assess the robustness of their product introduction decisions with respect to plausible follower scenarios, one of which involves predation. As such, an optimal strategy against a predatory follower provides a worst-case bound for the leader, which is currently missing from the literature in the context of product introduction.

In this article, we propose a hierarchical integer programming formulation to prescribe an optimal product introduction strategy for a leader firm in the presence of a predatory follower firm. Although integer programming formulations for product introduction do appear in the literature in monopolistic settings ([11, 12, 31, 32, 45]), ours is, to the best of our knowledge, the first exact optimization approach to a competitive product introduction problem.

Our general approach has been adopted in the context of a game theoretic network interdiction/fortification problem ([28, 41]). In their discrete network interdiction model, Lim and Smith [28] formulate a bilevel program in which integer variables are present only in the higher level optimization problem. This problem structure affords a penalty reformulation to circumvent nonlinearity when the bilevel program is transformed into an equivalent single level integer programming problem. This is extended to a three-stage network fortification problem by Smith et al. [41] in which a continuous bilinear program constitutes the lower level optimization problem while integer variables are confined within the higher level optimization problem.

By contrast, our multilevel optimization problem inevitably includes integer variables in the lower level optimization problem, and hence, requires a different approach than the ones pursued in [28] and [41]. Also, we demonstrate in this article that traditional cutting planes generated by a Benders-type process can be weak. Thus, we contribute a method of reformulating the master problem to generate cuts in an expanded variable space that are much stronger than the typical cuts that would be employed in such an algorithm. (In fact, we demonstrate that our new cuts in the expanded master problem variable space are capable of implying an exponential number of cutting planes in the original master variable space.)

The product introduction game analyzed in this article falls into a broad literature most commonly known as the economics and management of product variety. This is an interdisciplinary area that has attracted researchers from operations management, marketing, economics, and engineering disciplines. For a broad understanding of the area, the reader is referred to [19,22,24,36]. Here, we first give a brief overview of the variety literature in which predation has hardly ever been studied. We then review a few closely related articles in more detail.

Product variety researchers from economics and marketing disciplines have been predominantly occupied with socially optimal variety, the variety provided in various monopolistic or competitive market settings, and ways of rectifying a possible discrepancy between the two [24, 37]. Their models tend to be small-size and oriented toward generating big picture insights ([5, 8]). Unlike the economics literature, the marketing literature has also witnessed decision-aid models that have made a mark on industrial practice ([18]). In operations management, a major portion of product variety research has concentrated on the question of measuring, quantifying, and containing the negative effects of product variety on operations [17,26,43]. There is also a growing literature that focuses on operations-marketing interface issues such as variety and price competition between a custom and a standard product manufacturer [2]; variety, capacity, and production quantity decisions of a firm under free entry and a given set of potential product designs [38]; interaction of product line, pricing and make-to-order/make-to-stock decisions [12]; and optimal retail assortment that balances market gains and inventory costs due to variety [15,29,44].

One important feature of our work—sequential product introduction—has been captured in various ways in product variety literature. For instance, motivated by store opening decisions of multi-store competitors (e.g. two coffee shop chains in an urban area), Dasci and Laporte [10] study two self-interested firms locating stores sequentially. They do not analyze predatory behavior, and more importantly, they use a spatial choice model based on Hotelling [20]. Although their geographical location model can also be interpreted to take place in a product space, their approximate solution method is indeed more appropriate for geographical location problems. Chawla et al. [9] study a phenomenon similar to our article, in that there is a leader and a predatory follower engaged in a multistage game. But, unlike in our model, their leader and follower introduce one product at a time taking turns for a finite number of times. Furthermore, their consumer choice process is restricted to spatial models ala Hotelling [20], whereas our model can accommodate a wide variety of discrete choice models. The primary objective of Chawla et al. is to establish an upper bound for the first mover disadvantage. In contrast, we are interested in developing a solution methodology for the optimal actions by both parties.

The rest of this article is organized as follows. We formally introduce the new product introduction problem under predation in Section 2. In Section 3, we develop exact cutting plane solution methodologies for this problem by considering a multistage reformulation. In Section 4, we develop a set of heuristic approaches to the problem. We test the efficacy of these approaches in Section 5, and conclude the article in Section 6 with a summary of our work and directions for future research.

2. PROBLEM STATEMENT AND FORMULATION

We begin this section by presenting a formal problem statement in Section 2.1. The problem is formulated as a bilevel mixed-integer program in Section 2.2, with a brief note regarding the computational complexity of the problem.

2.1. Notation and Formal Statement

We consider a two-person Stackelberg game played by two companies who can potentially introduce products from the same set of (relatively homogeneous) product designs, N, into a market. One firm, the leader, first enters the market by introducing a set of products that belong to N. Then the other firm, the follower, enters the market by producing its line of products that also belong to N. The market is assumed to be exclusively divided into a set of segments, M. Segment $i \in M$ has an (ordered) product preference list, $O_i = (p_i^1, \dots, p_i^{k(i)})$, where $p_i^j \in N$, $\forall j = 1, \dots, k(i)$, are arranged such that segment i will purchase the first product (one having the smallest index j) that is available in the list. That is, if products $p_i^1, \dots, p_i^k$, $k < k(i)$, have not been introduced to the market, while product p_i^{k+1} has been, then consumers in market segment i will select product p_i^{k+1}. Furthermore, if no products in O_i have been introduced in the market, then segment i will not purchase any product. It is worth noting that the construct of preference lists have been successfully used in an actual industry application on rationalizing product lines [45] and in a theory of inventory competition between multiple firms each offering a single substitutable product [30]. Besides, because we do not impose any particular structure on the preference lists, they can come from a wide variety of discrete choice models. For instance, Hotelling's horizontal product differentiation model would require—once consumer locations are discretized into a finite number of market segments—each market segment to have a preference list in increasing order of transportation cost (disutility of consuming a less-than-ideal product). Random utility models such as the Multinomial Logit Model can also be incorporated, as long as the preference lists are constructed from N, and not from only the products introduced to the market.

For each $j \in N$, the leader and follower incur fixed costs b_j and c_j, respectively, for introducing products to the market. By assigning a prohibitively high fixed cost, a firm can be precluded from introducing a particular product (say due to technological know-how differences). Also, by assigning a zero fixed cost, we can model existing products currently offered by an incumbent (e.g., American Airlines). We assume that the leader has a budget of B, and the follower a budget of C, in the total expenditure for product introduction. Furthermore, assume that the revenue, r_{ij}, yielded from each market segment $i \in M$ consuming product $j \in N$ is known *a priori*. These revenue figures can be obtained from multiplying the number of consumers in a market segment by those consumers' willingness-to-pay for a particular product. Additionally, after the follower's penetration into the market, if both the leader and the follower offer product j, then we assume that $100\rho_{ij}\%$ of the demand for product j by market segment i is supplied by the leader's product, while the follower satisfies the rest. Although it is possible that players' introduction strategies may influence revenue shares between the leader and the follower, we assume, in our model, that inter-firm effects are more prominent than product line effects. That is, ρ_{ij}-values are constants and reflect the aspects of consumer choice that are not influenced directly by the product on offer, such as brand image, and they can be projected from the past market share data.

Note that the set of potential product designs and the definition of market segments (i.e., preference lists and revenues) are exogenous to our model, and they are assumed common knowledge. This simplification allows us to assume away market segmentation and pricing issues so that we can focus only on the product selection decision and its strategic implications. For some industries such as airlines, there are highly well-defined market segments, rendering our model more applicable. Furthermore, the model is suitable for situations where firms typically have a finite number of target prices instead of viewing the price as a continuous variable (e.g., by including in N the same flight more than once at different ticket prices).

Figure 1 illustrates this problem via an example with four product types. The thickest lines connect a market segment to a product representing the highest preference for that segment. Then, the second- and third-highest preferences are displayed using thick dotted and thin solid lines, respectively, between segments and products. For example, segment 3 seeks product 3 first, then product 2, and then product 4. If products 2, 3, and 4 have not been introduced, then market segment 3 will not purchase any product. Suppose that the leader introduces products 1 and 3, and the follower introduces products 3 and 4 as depicted in Fig. 1b. Assume a

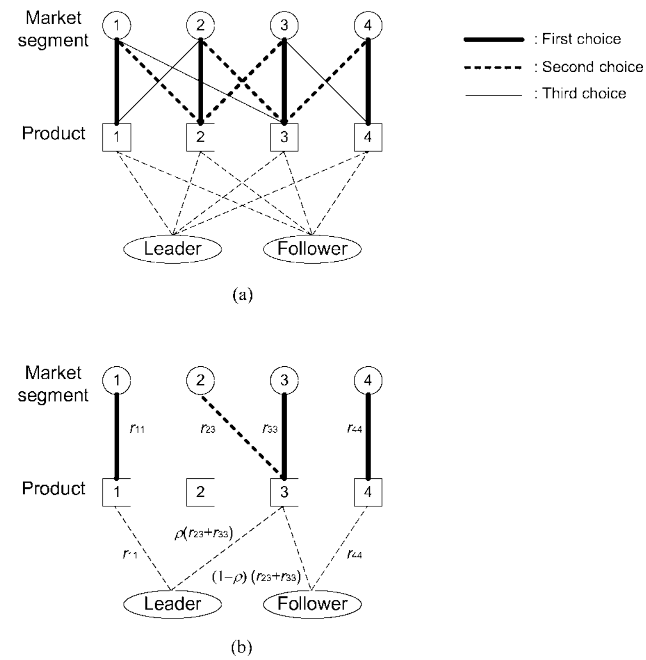

Figure 1. Illustration of the product introduction problem. (a) Example with four product types. (b) Market realization.

common market share factor $\rho_{ij} = \rho$, $\forall i = 1, \ldots, 4$ and $\forall j = 1, \ldots, 4$. Because product 1 is only supplied by the leader, the leader satisfies all demand from segment 1. Likewise, the follower will supply all demand for segment 4. However, both players introduce product 3, which is selected by both segments 2 and 3, because no player develops product 2. In this case, the leader will absorb $100\rho\%$ of the demand from segments 2 and 3.

As another example, suppose that $|M| = |N| = 2$, and the budgets are set so that the leader and follower can introduce one product each. Segment 1 prefers product 1 only and is worth \$100, and segment 2 prefers product 2 only and is worth \$91 (i.e., $O_1 = \{1\}$, $O_2 = \{2\}$, $r_{11} = 100$, and $r_{22} = 91$). However, $\rho_{11} = 0.1$ and $\rho_{22} = 0.95$, indicating that segment 2 is loyal to the leader while segment 1 is not. In this case, the leader should introduce product 2: if the competitor introduces product 1, the leader will earn \$91, while if the competitor also introduces product 2, the leader will earn \$86.45; hence, the leader is guaranteed at least \$86.45. One important note here distinguishes the difference between a *predatory* and a *self-interested* follower (i.e., a follower that only tries to maximize its own profit). If the leader feels that the follower is self-interested, then the leader would choose to instead introduce product 1: if the follower chooses to introduce product 1, the follower earns \$90 (and the leader only \$10), and if the follower chooses to introduce product 2, the follower earns \$91 (and the leader earns \$100). However, making the assumption of follower self-interest in this situation puts the leader in an extremely precarious position.

If the follower finds a slightly suboptimal solution (settling for \$90 instead of \$91), or assesses the worth of segment 2 as \$89 instead of \$91, then the leader's revenues will plummet to just \$10. It is therefore critical for companies to, at the very least, assess their level of vulnerability to predatory behavior.

2.2. Problem Formulation and Complexity

The objective of our problem is to maximize the leader's profit under the worst case scenario in which the follower acts as a predator. That is, the follower minimizes the leader's revenue by introducing a suitable set of products. To formulate the problem, let x_j and y_j, for all $j \in N$, denote binary variables of the leader and the follower, respectively. If $x_j = 1$ ($y_j = 1$), then the leader (follower) introduces product j. Else, product j is not offered by the respective players. Furthermore, let w_{ij} denote the leader's actual revenue obtained from market segment $i \in M$ that buys product $j \in O_i$ after the follower's penetration. The decision-making takes two stages in which the leader and the follower introduce products in turn, as modelled by the following maximin formulation.

$$\max_{x \in X} -\sum_{j \in N} b_j x_j$$

$$+\min \quad \sum_{i \in M} \sum_{j \in O_i} w_{ij} \tag{1a}$$

$$\text{s.t.} \quad w_{ij} \geq r_{ij}\left[x_j - (1-\rho_{ij})y_j - \sum_{k \in H_{ij}} (x_k + y_k)\right]$$

$$\forall i \in M,\ j \in O_i \tag{1b}$$

$$w_{ij} \geq 0 \quad \forall i \in M,\ j \in O_i \tag{1c}$$

$$\sum_{j \in N} c_j y_j \leq C \tag{1d}$$

$$y_j \text{ binary} \quad \forall j \in N, \tag{1e}$$

where $X = \{x : \sum_{j \in N} b_j x_j \leq B,\ x_j \text{ binary } \forall j \in N\}$, and where H_{ij} is the subset of O_i consisting of all products that have a higher preference than product j for segment i.

Given the set of products introduced by the leader, the follower's problem is to find a y-vector that minimizes the leader's revenue. Because the second stage problem is a minimization problem, we only need to impose lower bounds on the w-variables. Constraints (1b) state that w_{ij} is potentially positive when the leader supplies product j, and when j is the best available product for segment i (i.e., $x_j = 1$ and $\sum_{k \in H_{ij}} (x_k + y_k) = 0$). Additionally, if the follower does not produce product j ($y_j = 0$), then the leader gains a revenue of r_{ij} from market segment i. If both players introduce product j (and no higher-preference products are produced for that segment), then the leader gains a revenue of $\rho_{ij} r_{ij}$. In other cases, such as when $x_j = 0$ or $\sum_{k \in H_{ij}} (x_k + y_k) \geq 1$, we have that $w_{ij} = 0$ because the right-hand-side of (1b) is nonpositive, and the minimization objective forces w_{ij} to its lower bound of zero given by (1c).

REMARK 1: The follower's problem is NP-hard in the strong sense, and thus so is the leader's problem. Indeed, the leader's problem is not known to belong to NP, because evaluating the objective function value of a proposed solution to the leader's problem requires the optimization of the follower's problem.

We briefly describe a transformation from EXACT COVER BY THREE SETS (X3C) [14] to a decision version of the follower's problem. In X3C, there exists a set of elements $\{1, \ldots, 3p\}$, along with $q > p$ subsets, $S_1, \ldots, S_q$, of these elements, each having cardinality three. Our transformation creates a market segment set $M = \{1, \ldots, 3p\}$ and a product set $N = \{1, \ldots, 3p + q\}$, with $\rho_{ij} = 1/2$, $\forall i \in M,\ j \in N$. The preference lists O_i contain product i as the least-preferred product, $\forall i \in M$, and all products $3p + j$ such that $i \in S_j$ (in any arbitrary position of O_i but the last one). Suppose that the leader has introduced products $1, \ldots, 3p$, and that the follower has a budget of $C = p$, with $c_j = 1\ \forall j \in N$. Then, the follower can reduce the leader's revenue to zero if and only if a set of p products are introduced that have a higher preference than the leader's products, for all market segments. This is possible only by choosing p products in $\{3p+1, \ldots, 3p+q\}$ that collectively supersede the leader's products, which is equivalent to finding a solution to X3C. Interestingly, this transformation indicates that the follower's problem is *NP-hard*, even when the follower's budgetary restriction for product introduction is a simple cardinality constraint.

Difficulties in solving problem (1) by mathematical programming techniques arise due to the facts that the leader variables appear in constraints of the follower's problem, and that the follower's problem contains integer variables. In the following section, we design an exact algorithm to solve problem (1) by converting it to a three-stage problem and optimizing by a cutting plane approach.

REMARK 2: Before proceeding to the next section, we remark here that the model in (1) can also be applicable to the case where the leader is a new entrant to an industry and the follower is a powerful, established incumbent. In this case, the only difference is that the follower, being an incumbent firm, already has some products in place. That is, the follower's problem is to revise its product portfolio with the express purpose of driving the leader out of business (e.g., as American allegedly introduced its own flights for executives to compete with Legend). This revision may involve withdrawing some products and introducing new ones. The model in (1) can handle this by simply setting the fixed cost of introducing the existing products to zero (if withdrawal is costless). (If there is a cost to withdraw a product, then we can replace the corresponding term in the left-hand-side of constraint (1d) with $c_j(1-y_j)$, where c_j would be the cost to withdraw an existing product j.)

3. EXACT SOLUTION METHOD

We reformulate problem (1) as a problem amenable to decomposition in Section 3.1, and develop a cutting plane algorithm for its solution in Section 3.2. Finally, in Section 3.3, we develop an alternative modeling approach that permits the generation of stronger cutting planes, at the expense of a larger master problem formulation.

3.1. Problem Reformulation

In lieu of the two-stage formulation in (1), which is more directly interpretable in terms of new product introduction under predation, we transform the problem to a three-stage problem in a max-min-max framework. Each successive problem in this framework tackles, respectively, the leader's product introduction decisions, the follower's product introduction decisions, and the leader's revenue given both sets of decisions. Recall that in problem (1), constraints (1b) and (1c), together with the minimization of the objective function, determine the leader's revenue. Instead, we use the following maximization problem that determines the leader's revenue w_{ij} given leader decisions, $\bar{x}$, and follower decisions, $\bar{y}$.

$$\text{Maximize} \quad \sum_{i\in M}\sum_{j\in O_i} w_{ij} \tag{2a}$$

$$\text{subject to} \quad w_{ij} \le r_{ij}(1-\bar{x}_k) \quad \forall i\in M,\ j\in O_i,\ k\in H_{ij} \tag{2b}$$

$$w_{ij} \le r_{ij}(1-\bar{y}_k) \quad \forall i\in M,\ j\in O_i,\ k\in H_{ij} \tag{2c}$$

$$w_{ij} \le r_{ij}\bar{x}_j \quad \forall i\in M,\ j\in O_i \tag{2d}$$

$$w_{ij} \le r_{ij}(1-(1-\rho_{ij})\bar{y}_j) \quad \forall i\in M,\ j\in O_i. \tag{2e}$$

Because (2a) maximizes the sum of w-variables, it is sufficient to enforce only upper bounds on w_{ij} in (2b)–(2e). Constraints (2b)–(2c) guarantee that $w_{ij}=0$ when any product having a higher preference to market segment i than product j is introduced. Furthermore, $w_{ij}>0$ only when $x_j=1$ as enforced by constraint (2d), and constraint (2e) further restricts w_{ij} to be no more than $\rho_{ij}r_{ij}$ when the follower also introduces product j. We can now rewrite model (1) as a three-stage problem, in which formulation (2) serves as the innermost problem:

$$\underset{x\in X}{\text{Max}} - b^T x + \underset{y\in Y}{\min} \quad \max \sum_{i\in M}\sum_{j\in O_i} w_{ij} \tag{3a}$$

$$\text{s.t.} \quad \text{constraints (2b)} - \text{(2e)}, \tag{3b}$$

where $Y=\{y:\sum_{j\in N} c_j y_j \le C,\ y_j \text{ binary}, \forall j\in N\}$.

Note that the feasible region of problem (2) is nonempty and that the optimal objective function value is bounded, and so the optimal primal and dual solutions to (2) will have the same optimal objective value. Letting α_{ijk}, β_{ijk}, λ_{ij}, and μ_{ij} denote dual variables associated with constraints (2b), (2c), (2d), and (2e), respectively, the dual to (2) is given as:

$$\text{Minimize} \quad \sum_{i\in M}\sum_{j\in O_i}\sum_{k\in H_{ij}} \left[r_{ij}(1-\bar{x}_k)\alpha_{ijk} + r_{ij}(1-\bar{y}_k)\beta_{ijk}\right] + \sum_{i\in M}\sum_{j\in O_i}\left[r_{ij}\bar{x}_j\lambda_{ij} + r_{ij}(1-(1-\rho_{ij})\bar{y}_j)\mu_{ij}\right] \tag{4a}$$

$$\text{subject to} \quad \sum_{k\in H_{ij}}(\alpha_{ijk}+\beta_{ijk}) + \lambda_{ij} + \mu_{ij} = 1 \quad \forall i\in M,\ j\in O_i \tag{4b}$$

$$\alpha_{ijk}\ge 0 \quad \forall i\in M,\ j\in O_i,\ k\in H_{ij} \tag{4c}$$

$$\beta_{ijk}\ge 0 \quad \forall i\in M,\ j\in O_i,\ k\in H_{ij} \tag{4d}$$

$$\lambda_{ij}\ge 0 \quad \forall i\in M,\ j\in O_i \tag{4e}$$

$$\mu_{ij}\ge 0 \quad \forall i\in M,\ j\in O_i. \tag{4f}$$

Using this dual, the problem in (3) can be rewritten as follows.

$$\underset{x\in X}{\text{Max}} - b^T x + \min \sum_{i\in M}\sum_{j\in O_i}\sum_{k\in H_{ij}}\left[r_{ij}(1-x_k)\alpha_{ijk} + r_{ij}(1-y_k)\beta_{ijk}\right] + \sum_{i\in M}\sum_{j\in O_i}\left[r_{ij}x_j\lambda_{ij} + r_{ij}(1-(1-\rho_{ij})y_j)\mu_{ij}\right] \tag{5a}$$

$$\text{s.t.} \quad \text{constraints (4b)} - \text{(4f)} \tag{5b}$$

$$y\in Y. \tag{5c}$$

Given x, the inner minimization problem in (5) is a bilinear integer program in which bilinear terms $y_k\beta_{ijk}$ and $y_j\mu_{ij}$ consist of one binary variable and one continuous variable. Furthermore, note that variables α, β, λ, and μ are constrained by (4b)–(4f), whereas the y-variables are (disjointly) constrained by the set Y. Because fixing y yields a linear program associated with $(\alpha,\beta,\lambda,\mu)$, there exists an optimal solution $(\bar{y},\bar{\alpha},\bar{\beta},\bar{\lambda},\bar{\mu})$ such that $\bar{y}$ is an element of the finite set Y and

$(\bar{\alpha}, \bar{\beta}, \bar{\lambda}, \bar{\mu})$ is an extreme point of the polyhedral set defined by constraints (4b)–(4f). Let Π denote the set of extreme points to the polyhedron defined by (4b)–(4f). Then, we can rewrite (5) as:

$$\begin{aligned}\max_{x \in X} &- b^T x \\ &+ \min_{y \in Y,\, (\alpha,\beta,\lambda,\mu) \in \Pi} \sum_{i \in M} \sum_{j \in O_i} \sum_{k \in H_{ij}} \big[r_{ij}(1 - x_k)\alpha_{ijk} \\ &\qquad + r_{ij}(1 - y_k)\beta_{ijk} \big] \\ &\qquad + \sum_{i \in M} \sum_{j \in O_i} \big[r_{ij} x_j \lambda_{ij} \\ &\qquad + r_{ij}(1 - (1 - \rho_{ij}) y_j)\mu_{ij} \big]. \quad (6)\end{aligned}$$

REMARK 3: Even if the inner minimization problem in (5) contains y-variables that are not restricted to be binary, our strategy of linearizing the $y_k \beta_{ijk}$ and $y_j \mu_{ij}$ terms in the objective is still possible by noting that there exists an optimal solution to (5) in which all α, β, λ, and μ are binary-valued. We could then enforce binary restrictions on the α, β, λ, and μ variables and linearize as usual. If, perhaps in a different application, linearization is not possible, we could instead resort to more computationally intensive techniques that can handle broader classes of bilinear programs. For instance, we could use concavity cuts or implicit enumeration of basic feasible solutions ([1, 3, 23, 39]), or explore computational algebraic geometry approaches such as the generation of Gröbner bases of constraint (and objective) polynomials ([7, 42]) and the construction of an equivalent positive semidefinite program ([25]).

3.2. Cutting Plane Algorithm

Note that Π and Y are finite sets. Furthermore, because the objective function in the inner minimization problem of (6) is affine in variables x, we can use a cutting plane algorithm in the spirit of Benders decomposition. The relaxed master problem (RMP) for a given set Θ of potential solutions to the inner minimization problem takes on the form

$$\begin{aligned}\text{maximize} \quad & -b^T x + z && (7a)\\ \text{subject to} \quad & z \le \sum_{i \in M} \sum_{j \in O_i} \sum_{k \in H_{ij}} \big[r_{ij}(1 - x_k)\bar{\alpha}_{ijk} \\ & \qquad + r_{ij}(1 - \bar{y}_k)\bar{\beta}_{ijk} \big] \\ & \qquad + \sum_{i \in M} \sum_{j \in O_i} \big[r_{ij} x_j \bar{\lambda}_{ij} \\ & \qquad + r_{ij}(1 - (1 - \rho_{ij})\bar{y}_j)\bar{\mu}_{ij} \big] \\ & \qquad \forall (\bar{y}, \bar{\alpha}, \bar{\beta}, \bar{\lambda}, \bar{\mu}) \in \Theta \subseteq Y \times \Pi && (7b)\\ & x \in X, && (7c)\end{aligned}$$

while the subproblem is given by the inner minimization problem in (5). An overview of our cutting plane algorithm is described as follows.

Cutting Plane Algorithm
Initialization. Set $\Theta = \emptyset$, $LB = 0$, and $x_j^\star = 0\ \forall j \in N$.
Step 1. Find an optimal solution $(\bar{x}, \bar{z})$ to RMP.
Step 2. If $LB = \bar{z} - b^T \bar{x}$, then $x^\star$ is optimal, and the algorithm terminates. Otherwise, given $\bar{x}$, find an optimal solution $\bar{y} \in Y$ and $(\bar{\alpha}, \bar{\beta}, \bar{\lambda}, \bar{\mu}) \in \Pi$ to the subproblem given by (5). (We discuss solution methods for the subproblem below.) Let $v(\bar{x})$ denote the optimal objective value of this subproblem.
Step 3. If $v(\bar{x}) - b^T \bar{x} > LB$, put $LB = v(\bar{x}) - b^T \bar{x}$ and $x^\star = \bar{x}$.
Step 4. Add $(\bar{y}, \bar{\alpha}, \bar{\beta}, \bar{\lambda}, \bar{\mu})$ to Θ, and return to Step 1.

At each step of the algorithm, we keep track of our incumbent solution, $x^\star$, with objective value LB. In Step 1, we solve the RMP, which yields an upper bound of $\bar{z} - b^T \bar{x}$ on the optimal objective value of the problem. The algorithm terminates when the lower bound equals to the upper bound. While there exists a gap between the lower and upper bounds, we compute the objective value of the current problem solution, $v(\bar{x}) - b^T \bar{x}$, by solving the subproblem to obtain the follower's reaction to $\bar{x}$ and the resulting revenue achieved. The incumbent solution is then updated in Step 3 if the objective function value exceeds LB. Because the current solution $(\bar{x}, \bar{z})$ violates some constraints having the form of (7b) that is not currently included in RMP, we add $(\bar{y}, \bar{\alpha}, \bar{\beta}, \bar{\lambda}, \bar{\mu})$ to Θ, which induces a most-violated cutting plane to RMP, thus avoiding the regeneration of the prior solution $(\bar{x}, \bar{z})$.

The subproblem in the cutting plane algorithm above is a mixed integer bilinear program, which can be linearized as follows. First, note that (4b)–(4f) implies that all α-, β-, λ-, and μ-variables are bounded above by one. Substituting $\gamma_{ijk} = y_k \beta_{ijk}$ and $\xi_{ij} = y_j \mu_{ij}$, and incorporating linearization constraints to enforce this relationship, the subproblem can be written as follows.

$$\begin{aligned}\text{Minimize} \quad & \sum_{i \in M} \sum_{j \in O_i} \sum_{k \in H_{ij}} \big[r_{ij}(1 - x_k)\alpha_{ijk} \\ & \qquad + r_{ij}(\beta_{ijk} - \gamma_{ijk}) \big] \\ & \qquad + \sum_{i \in M} \sum_{j \in O_i} \big[r_{ij} x_j \lambda_{ij} \\ & \qquad + r_{ij}(\mu_{ij} - (1 - \rho_{ij})\xi_{ij}) \big] && (8a)\\ \text{subject to} \quad & \text{constraints (4b)} - \text{(4f)} && (8b)\\ & y \in Y && (8c)\\ & \gamma_{ijk} \le y_k \quad \forall i \in M,\ j \in O_i,\ k \in H_{ij} && (8d)\\ & \gamma_{ijk} \le \beta_{ijk} \quad \forall i \in M,\ j \in O_i,\ k \in H_{ij} && (8e)\\ & \xi_{ij} \le y_j \quad \forall i \in M,\ j \in O_i && (8f)\\ & \xi_{ij} \le \mu_{ij} \quad \forall i \in M,\ j \in O_i. && (8g)\end{aligned}$$

Because bilinear terms appear in the objective function with negative coefficients, only upper bounding constraints on γ_{ijk} and ξ_{ij} are enforced in (8d)–(8g), and the typical lower bounding constraints $\gamma_{ijk} \geq 0$, $\gamma_{ijk} \geq y_k + \beta_{ijk} - 1$, $\forall i \in M,\ j \in O_i,\ k \in H_{ij}$, and $\xi_{ij} \geq 0$, $\xi_{ij} \geq y_j + \mu_{ij} - 1$, $\forall i \in M,\ j \in O_i$, are omitted.

REMARK 4: An alternative way to find a solution to the subproblem is rendered by the original formulation in (1). Given x, the inner minimization problem is a mixed integer linear program, which is smaller than the linearized problem (8). Then, given $\bar{y}$ obtained by solving this problem, the solution of the linear program in (4) will yield an optimal set of variable values $(\bar{\alpha}, \bar{\beta}, \bar{\lambda}, \bar{\mu})$ to the subproblem. We will experiment with both approaches to solving the subproblem in our computational study.

Note that for each $i \in M$, $j \in O_i$, (4) is a separable continuous knapsack problem given $\bar{x}$ and $\bar{y}$. This problem can then be solved by examining each pair $i \in M$, $j \in O_i$, and seeking the minimum of the terms $r_{ij}(1-\bar{x}_k)$ for each $k \in H_{ij}$ (corresponding to α_{ijk}), $r_{ij}(1-\bar{y}_k)$ for each $k \in H_{ij}$ (corresponding to β_{ijk}), $r_{ij}\bar{x}_j$ (corresponding to λ_{ij}), and $r_{ij}(1-(1-\rho)\bar{y}_j)$ (corresponding to μ_{ij}). We would then set a dual variable corresponding to the minimum of these terms equal to one, and all other dual values equal to zero for that i, j pair. Note that several ties may exist when examining the terms for each i, j pair as mentioned above, particularly when several products $k \in H_{ij}$ have been introduced by the leader and/or follower (corresponding to several choices of α_{ijk}- and β_{ijk}-variables that can take on positive values). The implications of such ties are noted in the next subsection, where we examine methods for strengthening the cutting planes (7b) passed from the subproblem.

REMARK 5: Suppose that the leader takes advantage of a monopoly for some period of time because the follower's action is relatively slow (for example, due to a lack of new technology or patent constraints). Define $0 \leq \sigma < 1$ to be the proportion of the time horizon during which no competition exists. This extension can be easily adopted by rewriting the problem in (1) as follows.

$$\text{Maximize} \quad -\sum_{j\in N} b_j x_j + \sum_{i\in M}\sum_{j\in O_i} v_{ij} + z(x) \tag{9a}$$

$$\text{subject to} \quad v_{ij} \leq \sigma r_{ij}(1-x_k) \quad \forall i \in M,\ j \in O_i,\ k \in H_{ij} \tag{9b}$$

$$v_{ij} \leq \sigma r_{ij} x_j \qquad i \in M,\ j \in O_i \tag{9c}$$

$$\sum_{j\in N} b_j x_j \leq B, \quad x_j \text{ binary } j \in N \tag{9d}$$

$$x_j \text{ binary} \qquad j \in N, \tag{9e}$$

where

$$z(x) = \text{minimize} \quad \sum_{i\in M}\sum_{j\in O_i} (1-\sigma) w_{ij} \tag{10a}$$

$$\text{subject to} \quad \text{constraints (1b)} - \text{(1e)}. \tag{10b}$$

The cutting plane algorithm discussed earlier can then be applied with obvious modifications. Moreover, as σ tends towards a value of one, we anticipate that fewer cutting planes will need to be added to the model, because the precompetition portion of the objective will tend to dominate the effects of the follower's action.

3.3. Alternative Modeling Strategy

The cutting planes (7b) can be interpreted as the statement of an upper bound on z, which can be modified by the selection or deselection of certain x-variables associated with positive α- and λ-values. The selection of these dual variables thus guides the decomposition search procedure by encouraging the selection of x-variables that maximize z. Hence, setting $\alpha_{ijk} = 1$ for some $i \in M$, $j \in O_i$, and $k \in H_{ij}$ in the solution of (4) yields a term that implies that setting $x_k = 0$ can increase the right-hand-side by r_{ij}. However, if there were at least two terms $k_1, k_2 \in H_{ij}$ such that $\bar{x}_{k_1} = \bar{x}_{k_2} = 0$ in the previous solution, then setting either $\alpha_{ijk_1} = 1$ or $\alpha_{ijk_2} = 1$ does not accurately convey the condition that both x_{k_1} and x_{k_2} must equal to zero to possibly capture segment i by introducing product j.

The existence of alternative optimal dual solutions in this case implies that several cuts can be generated from each $\bar{x}$-solution passed to the subproblem. Worse, we will demonstrate below that there may exist an exponential number of these cuts, and that these cuts do not dominate one another. Because the source of this difficulty appears to lie in the need to express conditions on certain combinations of x-variables, we consider the following reformulation of the master problem. For each segment/product pair $i \in M$, $j \in O_i$, define variables u_{ij} to equal one if j is the highest-preference product for market segment i that has been introduced by the leader, and zero otherwise. That is, u_{ij} equals to $x_j \prod_{k\in H_{ij}}(1-x_k)$. We linearize each of these terms using the following relationships:

$$u_{ij} \leq x_j \tag{11a}$$

$$u_{ij} \leq (1-x_k) \quad \forall k \in H_{ij} \tag{11b}$$

$$u_{ij} \geq x_j + \sum_{k\in H_{ij}}(1-x_k) - |H_{ij}| \tag{11c}$$

$$u_{ij} \geq 0. \tag{11d}$$

Problem (2) then becomes

$$\text{maximize} \quad \sum_{i\in M}\sum_{j\in O_i} w_{ij} \tag{12a}$$

$$\text{subject to} \quad w_{ij} \le r_{ij}\bar{u}_{ij} \quad \forall i \in M,\ j \in O_i \tag{12b}$$

$$w_{ij} \le r_{ij}(1-\bar{y}_k) \quad \forall i \in M,\ j \in O_i,\ k \in H_{ij} \tag{12c}$$

$$w_{ij} \le r_{ij}(1-(1-\rho_{ij})\bar{y}_j) \quad \forall i \in M,\ j \in O_i, \tag{12d}$$

and taking its dual (with α_{ij} now associated with (12b), β_{ijk} associated with (12c), and μ_{ij} associated with (12d)) now yields:

$$\text{Minimize} \quad \sum_{i\in M}\sum_{j\in O_i}\Bigg[r_{ij}\bar{u}_{ij}\alpha_{ij} + \Bigg[\sum_{k\in H_{ij}} r_{ij}(1-\bar{y}_k)\beta_{ijk}\Bigg] + r_{ij}(1-(1-\rho_{ij})\bar{y}_j)\mu_{ij}\Bigg] \tag{13a}$$

$$\text{subject to} \quad \alpha_{ij} + \sum_{k\in H_{ij}}\beta_{ijk} + \mu_{ij} = 1 \quad \forall i \in M,\ j \in O_i \tag{13b}$$

$$\alpha_{ij} \ge 0 \quad \forall i \in M,\ j \in O_i \tag{13c}$$

$$\beta_{ijk} \ge 0 \quad \forall i \in M,\ j \in O_i,\ k \in H_{ij} \tag{13d}$$

$$\mu_{ij} \ge 0 \quad \forall i \in M,\ j \in O_i. \tag{13e}$$

The resulting cutting plane is given by

$$z \le \sum_{i\in M}\sum_{j\in O_i}\Bigg[r_{ij}u_{ij}\bar{\alpha}_{ij} + \Bigg[\sum_{k\in H_{ij}} r_{ij}(1-\bar{y}_k)\bar{\beta}_{ijk}\Bigg] + r_{ij}(1-(1-\rho_{ij})\bar{y}_j)\bar{\mu}_{ij}\Bigg]. \tag{14}$$

Once again, the derivation of duals to this problem, given $\bar{u}$ and $\bar{y}$, is done in a single pass through the objective terms. However, in this case, there exists a mechanism to break ties to obtain the strongest possible inequality (i.e., to minimize the right-hand-side of (14)). We will solve (13) by scanning through each pair of $i \in M$, $j \in O_i$, and first seeking a value of $r_{ij}(1-\bar{y}_k)$ that currently equals to zero. If there exists such a k, then we set $\beta_{ijk} = 1$ (any such choice of k yields the same inequality, because fixed y-values are passed in (14)). Otherwise, if $u_{ij} = 0$, then we select $\alpha_{ij} = 1$, and if not, we select $\mu_{ij} = 1$ if $\bar{y}_j = 1$. If none of these conditions hold, then we select $\alpha_{ij} = 1$, because the term $r_{ij}u_{ij}$ is less than or equal to the alternative terms $r_{ij}(1-\bar{y}_k)\ \forall k \in H_{ij}$ or $r_{ij}(1-(1-\rho_{ij})\bar{y}_j)$ (all of which equal to r_{ij}), which could be added in place of $r_{ij}u_{ij}$. In this way, a unique strongest cutting plane now exists for this problem given values of x and y (and hence u as well).

REMARK 6: It is possible that at some iteration of the cutting plane algorithm, there may exist an exponential number of nondominated cutting planes of the form (7b) that can be generated. However, if the alternative modeling strategy presented in this subsection is employed, then a single cutting plane (14), together with inequalities (11a) and (11b), may be capable of implying all of the exponentially-many cutting planes of the form (7b). For example, consider an instance in which $M = \{0,\dots,p\}$ and $N = \{0,\dots,3p\}$ for some $p \ge 2$, where $O_0 = \{0\}$, and $O_i = \{3i-2,\ 3i-1,\ 3i\}$, $\forall i = 1,\dots,p$. Let $r_{00} = 2p$ and $r_{ij} = 1$ for all other $i = 1,\dots,p$ and $j \in O_i$; also, let $\rho_{ij} = 1/2\ \forall i \in M, j \in O_i$. Suppose that the leader has introduced products $3i-2$ and $3i-1$, for all $i = 1,\dots,p$. If the follower has a budget of $C = 1$, with $c_j = 1$, $\forall j \in N$, then the follower will choose to introduce product 0, thus reducing the leader's profit by p. We must now compute the solution to (4) in the initial model, or to (13) in the alternative model.

In model (4), with respect to market segment 0, we must set $\mu_{00} = 1$. With respect to each market segment $i = 1,\dots,p$, we set $\lambda_{i,3i-2} = 1$, $\alpha_{i,3i-1,3i-2} = 1$, and then can set either $\alpha_{i,3i,3i-2}$ or $\alpha_{i,3i,3i-1}$ equal to one, with all other variables equal to zero. Define a binary indicator $\delta_i = 1$ if we decide to set $\alpha_{i,3i,3i-2} = 1$ and $\delta_i = 0$ if we decide to set $\alpha_{i,3i,3i-1} = 1$. Then the inequality obtained from this dual solution is:

$$z \le p + \sum_{i=1}^{p}(2 - \delta_i x_{3i-2} - (1-\delta_i)x_{3i-1}). \tag{15}$$

For each of the 2^p choices of δ_i, we obtain a different, nondominated inequality (15).

On the other hand, suppose we solve (13). We again set $\mu_{00} = 1$. For $i = 1,\dots,p$, we now set $\alpha_{i,3i-2} = \alpha_{i,3i-1} = \alpha_{i,3i} = 1$ to obtain the strongest possible inequality, given by

$$\begin{aligned} z &\le p + \sum_{i=1}^{p}(u_{i,(3i-2)} + u_{i,(3i-1)} + u_{i,(3i)}) \\ &\le p + \sum_{i=1}^{p}(x_{3i-2} + (1-x_{3i-2}) + u_{i,(3i)}) \\ &= p + \sum_{i=1}^{p}(1 + u_{i,(3i)}), \end{aligned} \tag{16}$$

where the second inequality is due to (11a) and (11b).

Consider an inequality (15) produced according to a set of choices $\hat{\delta}_i$, $\forall i = 1,\dots,p$, and note that (11b) includes the restrictions $u_{i,(3i)} \le (1-x_{3i-2})$ and $u_{i,(3i)} \le (1-x_{3i-1})$. Hence, by replacing $u_{i,(3i)}$ with $(1-x_{3i-2})$ when $\hat{\delta}_i = 1$, and with $(1-x_{3i-1})$ when $\hat{\delta}_i = 0$ (which are valid substitutions due to (11b)), we see that (16) implies (15). Moreover, because this operation can be done for any of the exponential choices of the δ-values, we have that (16), along with (11a) and (11b), implies an exponential number of cutting planes of the form (15).

4. HEURISTIC METHODS

Recall that the follower's problem itself is NP-hard, and thus computational efforts to solve even the follower's problem to optimality increase exponentially as the problem size increases (unless P = NP). Therefore, to provide practical solutions for such cases, we propose a heuristic approach to the problem in this section. Our approach is a relaxation-restriction method, in which a linear programming relaxation is used to solve the follower's problem. (Restriction is used here because a relaxation of the follower's problem results in a restriction of the leader's problem.)

For convenience of presentation, let us define $L_{ij} \subset O_i$, $\forall i \in M$, as the subset of elements that have a lower preference than product $j \in O_i$ for segment i. Then, fixing x as constant from (1), the follower's problem becomes the following linear programming problem:

$$\text{Minimize} \quad \sum_{i\in M}\sum_{j\in O_i} w_{ij} \tag{17a}$$

$$\text{subject to} \quad w_{ij} + r_{ij}(1-\rho_{ij})y_j + r_{ij}\sum_{k\in H_{ij}} y_k \geq r_{ij}\left[x_j - \sum_{k\in H_{ij}} x_k\right] \quad \forall i \in M, j \in O_i \tag{17b}$$

$$y_j \leq 1 \quad \forall j \in N \tag{17c}$$

$$\sum_{j\in N} c_j y_j \leq C \tag{17d}$$

$$w_{ij} \geq 0 \quad \forall i \in M, j \in O_i, \qquad y_j \geq 0 \quad \forall j \in N. \tag{17e}$$

Let $\tilde{\alpha}_{ij}$, $\tilde{\beta}_j$, and $\tilde{\gamma}$ denote dual variables associated with constraints (17b), (17c), and (17d), respectively. Then, the dual problem of (17) is given by

$$\text{maximize} \quad \sum_{i\in M}\sum_{j\in O_i} r_{ij}\left[x_j - \sum_{k\in H_{ij}} x_k\right]\tilde{\alpha}_{ij} - \sum_{j\in N}\tilde{\beta}_j - C\tilde{\gamma} \tag{18a}$$

$$\text{subject to} \quad \sum_{i:j\in O_i} r_{ij}\left[(1-\rho_{ij})\tilde{\alpha}_{ij} + \sum_{k\in L_{ij}}\tilde{\alpha}_{ik}\right] - \tilde{\beta}_j - c_j\tilde{\gamma} \leq 0 \quad \forall j \in N \tag{18b}$$

$$0 \leq \tilde{\alpha}_{ij} \leq 1 \; \forall i \in M,\; j \in O_i, \qquad \tilde{\beta}_j \geq 0 \quad \forall j \in N, \quad \tilde{\gamma} \geq 0. \tag{18c}$$

Note that problem (17) is feasible for any x. Also, its objective function value is bounded below because of nonnegativity of w. Hence, the optimal objective values of (17) and (18) are the same. Replacing the follower's problem with (18), and releasing x's as variables, the leader's problem becomes the following maximization problem:

$$\text{Maximize} \quad -\sum_{i\in N} b_j x_j + \sum_{i\in M}\sum_{j\in O_i} r_{ij}\left[x_j - \sum_{k\in H_{ij}} x_k\right]\tilde{\alpha}_{ij} - \sum_{j\in N}\tilde{\beta}_j - C\tilde{\gamma} \tag{19a}$$

$$\text{subject to} \quad \text{constraints (18b,c)} \tag{19b}$$

$$x \in X. \tag{19c}$$

Problem (19) is a mixed integer bilinear program in which bilinear terms are comprised of binary variable x and continuous variable $\tilde{\alpha}$, and appear only in the objective function. As discussed in the previous section, one can linearize these terms by substituting bilinear terms as $\tilde{\lambda}_{ij} = x_j\tilde{\alpha}_{ij}$ and $\tilde{\mu}_{ijk} = x_k\tilde{\alpha}_{ij}$, and adding the following linearization constraints: $\tilde{\lambda}_{ij} - \alpha_{ij} \leq 0$ and $\tilde{\lambda}_{ij} - x_j \leq 0, \forall i \in M,\; j \in O_i$; and $\alpha_{ij} + x_k - \tilde{\mu}_{ijk} \leq 1$ and $\tilde{\mu}_{ijk} \geq 0, \forall i \in M,\; j \in O_i,\; k \in H_{ij}$.

Although this relaxation-restriction method simplifies the problem by relaxing the follower's problem, it still requires a solution of a mixed integer programming problem, and hence this method may not be suitable for solving very large problems. For such problems, we recommend two different polynomial-time approaches. In the single relaxation-restriction method, we solve the linear programming relaxation of the leader's restricted problem, obtaining an optimal fractional solution $\hat{x}$. Then, we sort the x-variables in nonincreasing order of their solution values, and reindex them such that $\hat{x}_1 \geq \hat{x}_2 \geq \cdots \geq \hat{x}_{|N|}$. Next, we iterate from $j = 1, \ldots, |N|$, assigning a value of zero to variable x_j if b_j exceeds the remaining product introduction budget, and setting $x_j = 1$ otherwise while reducing the remaining product introduction budget by b_j.

An alternative strategy, called the multiple relaxation-restriction method, fixes a limited set of variable values to zero and one, and then updates and resolves the leader's restricted problem to provide a new fractional solution, until an integer feasible solution results. Specifically, this heuristic proceeds by solving the linear programming relaxation to the leader's restricted problem and obtaining an optimal solution $\hat{x}$. First, all variables x_j, $j \in N$: $\hat{x}_j = 1$, are fixed to one, and the remaining budget $\bar{B}$ is computed as B minus the sum of product introduction costs of all products $j \in N$ that have been fixed to one. Next, we fix $x_j = 0$ for all $j \in N$ such that $b_j > \bar{B}$. Let $F \subseteq N$ denote the set of free variables (those not yet fixed to zero or one). If $\hat{x}_j$ is binary for each $j \in F$, then terminate the algorithm. Else, also fix $x_p = 1$ for

$p \in \text{argmax}_{j \in F: \hat{x}_j < 1}\{\hat{x}_j\}$. (That is, p is the index of a largest fractional solution value among the free variables.) Remove p from F, decrease $\bar{B}$ by b_p, fix any other x_j to zero for $j \in F : b_j > \bar{B}$, and repeat. Because at least one variable is fixed at each iteration, no more than $|N|$ iterations of this algorithm will be performed before termination.

REMARK 7: We also investigated the development of an iterative construction heuristic for this problem in which the leader and follower move in consecutive turns by introducing one product at a time in response to the other. To anticipate which product would be introduced at each turn, we define the relative preference ω_{ij} for each market segment $i \in M$ and for each product $j \in N$, given a current x- and y-solution, as

$$\omega_{ij} = \begin{cases} f(k) & \text{if } j = p_i^k,\ j \in O_i, \\ & \quad \text{and } x_{p_i^h} = y_{p_i^h} = 0 \quad \forall h \in \{1, \ldots, k\} \\ 0 & \text{otherwise,} \end{cases} \tag{20}$$

where $f(k)$ is a nonincreasing function. Note that ω_{ij} represents the preference extent of market segment i toward product j. We accordingly define the weighted revenue for each product $j \in N$ as $\phi_j = \sum_{i \in M} r_{ij}\omega_{ij}\ \forall j \in N$.

In this heuristic, the leader and follower repeatedly introduce products based on the ratio of current weighted revenue to product introduction cost. We also experimented with allowing the leader and/or follower to introduce several products at once. However, this methodology proved ineffective in comparison to the relaxation-restriction methods, primarily because the sequential decision-making process fails to mirror the true nature of the Stackelberg game being modelled.

REMARK 8: When designing a heuristic method, it is desirable to derive a theoretical worst-case bound of its performance. Indeed, such bounds are available for some linear programming relaxation-based heuristics ([4, 27]). However, it is noteworthy that the worst-case bounds for our heuristic strategies cannot be derived using the approach in those references, because the final objective function value of our heuristics does not reflect the actual value of the problem. That is, given the heuristic solution, we still need to solve the follower's problem to evaluate the objective function value, which by itself, is NP-hard. Although this article primarily focused on the exact methods, we note that deriving theoretical worst-case bounds can be addressed as an interesting line of future research.

5. COMPUTATIONAL STUDY

In this section, we present a computational study for investigating the performance of the three exact and three heuristic methods presented earlier. Let CUT1, CUT2, and CUT3 denote the cutting plane algorithm proposed in 3.2, its variant as described in Remark 4, and the method using the alternative modeling strategy given in 3.3, respectively. For heuristics, we denote the relaxation-restriction method by RM, the single relaxation-restriction method by SRM, and the multiple relaxation-restriction method by MRM. We generated 22 sets of test instances, each of which contains 10 random instances, for a total of 220 test instances. The first 11 sets, S1–S11, have relatively small sizes that are intended for implementing exact algorithms as well as investigating the quality of our heuristic methods relative to optimality. Test sets L1–L11 contain larger instances that are suitable for comparing heuristic performance relative to each other. Specific characteristics of these test sets are summarized in Table 1. Common values for $|O_i|$ and ρ_{ij} (i.e., $|O_i| = |O|,\ \forall i \in N$ and $\rho_{ij} = \rho,\ \forall i \in N,\ j \in O_i$) are used for the purpose of a consistent performance measurement. Each test set is designed to have a distinctive characteristic. For example, S1, S2, and S3 are identical except for the values of $|M|$, which are $|N|/2$, $|N|$, and $3|N|/2$, respectively. Similarly, instance parameters of S2, S8, and S9 are different only in the values of budget ratios between the leader and the follower. Also,

Table 1. Summary of test set configurations.

Set	$\|N\|$	$\|M\|$	$\|O\|$	ρ	$B:C$	σ	Set	$\|N\|$	$\|M\|$	$\|O\|$	ρ	$B:C$	σ
S1	12	6	6	0.5	1:1	0	L1	60	30	30	0.5	1:1	0
S2	12	12	6	0.5	1:1	0	L2	60	60	30	0.5	1:1	0
S3	12	18	6	0.5	1:1	0	L3	60	90	30	0.5	1:1	0
S4	12	12	3	0.5	1:1	0	L4	60	60	15	0.5	1:1	0
S5	12	12	9	0.5	1:1	0	L5	60	60	45	0.5	1:1	0
S6	12	12	6	0.2	1:1	0	L6	60	60	30	0.2	1:1	0
S7	12	12	6	0.8	1:1	0	L7	60	60	30	0.8	1:1	0
S8	12	12	6	0.5	2:1	0	L8	60	60	30	0.5	2:1	0
S9	12	12	6	0.5	1:2	0	L9	60	60	30	0.5	1:2	0
S10	12	12	6	0.5	1:1	0.3	L10	28	28	14	0.5	1:1	0.3
S11	12	12	6	0.5	1:1	0.7	L11	28	28	14	0.5	1:1	0.7

note that instances in S10–S11 and L10–L11 have a positive value of σ, which allows the leader to enjoy a monopoly for this proportion of the time period, as described in Remark 5. Observing that the augmented problem (9)–(10) significantly increases the size of the mixed integer programming problem that needs to be solved by RM, the instances in L10–L11 are chosen to have moderate sizes ($|N| = 28$) compared to other large instances in L1–L9 ($|N| = 60$). Given the size specified in Table 1, each instance is created by randomly generating revenues (r_{ij}), product introduction costs (b_j and c_j), budgets (B and C), and preference lists (p_i^j).

More specifically, we first determine the size of each market segment by randomly choosing an integer value from the interval [50, 150]. Then, r_{ij} is computed by the size of market segment i multiplied by the price of product j, $\forall i \in M$, $j \in O_i$, which is again randomly selected from [10, 50]. Similarly, each product's introduction cost is obtained by multiplying 50 by its price. We use the same product introduction cost between the leader and the follower (i.e., $b_j = c_j \ \forall j \in N$). Then, B and C are computed so that $B + C$ is equal to the sum of all product introduction costs, and so that the ratio $B : C$ specified in Table 1 is satisfied. For each market segment $i \in M$, the most preferred product (i.e., p_i^1) is randomly selected among products that have not been assigned to the first place of any preference list that are previously generated. (If none exists, it is randomly selected from N.) Then, other elements of the list O_i are randomly selected from $N \backslash \{p_i^1\}$. All procedures are coded in C++ in conjunction with CPLEX 9.0 Concert Technology and run on a Dell Power Edge 2600 computer equipped with dual Pentium-4 3.2Ghz processors and 6 GB of memory.

The performance of the exact methods is presented in Table 2. Columns 2–4 display the average CPU time of 10 instances of the corresponding sets, while columns 5–7 report the average number of cuts generated until the algorithm finds an optimal solution. First, we remark that CUT3 clearly exhibits the best performance among implemented algorithms, while CUT2 is the second best. In particular, CUT3 was able to solve each test instance within ten seconds except for two instances (one in S3 and one in S6) for which CUT3 consumed 31.2 and 16.49 seconds. Overall, CUT3 consumed only 0.3% and 16.7% of average CPU times that were required by CUT1 and CUT2, respectively (see the last row of Table 2).

Comparing results for S1–S3, it is obvious that more CPU time and cutting planes are required as the number of segments ($|M|$) increases. For example, when CUT3 is used to solve instances in S3, the number of u-, α-, β-, and μ-variables are tripled when compared to those for solving instances in S1. This results in greater than a 1500% increase in CPU time and 120% increase in the number of cuts on average. Furthermore, when observing results of CUT3, increases in CPU time and in the number of cuts from S1 to S2 were 175% and 17%, respectively. However, these increases from S2 to S3 are amplified to 496% and 92%, respectively. Observe that CUT2 displays a similar pattern, while CUT1 does not. Next, the effect of different sizes of preference lists can be observed by looking at results for S4, S2, and S5 (Table 1). Methods CUT1 and CUT2 display an increasing pattern in both CPU time and the number of cuts as $|O_i|$ increases. The amount of CPU time consumed by CUT3 also increases as $|O_i|$ increases, but does not require an increase in the number of cuts.

Comparing results for S6, S2, and S7 yields the effect of the varying market share values (ρ). Interestingly, computational effort tends to decrease as the leader's market share increases. For example, CUT3 consumed 5.3 seconds on average for solving instances with $\rho_{ij} = 0.2$ (S6), but it used only 0.6 seconds when used to instances in S7, where $\rho_{ij} = 0.8$. Next, the effect of different budget ratios between the leader and the follower is observed from the results of S9, S2, and S8. The computational effort required by CUT1 and CUT2 increase as the ratio of the leader's budget to the follower's budget increases. However, note that CUT3 showed the opposite

Table 2. Average CPU time and number of cuts over 10 instances.

	Average CPU time (sec)			Average number of cuts		
Set	CUT1	CUT2	CUT3	CUT1	CUT2	CUT3
S1	122.9	2.1	0.4	348.7	56.8	19.8
S2	569.9	5.3	1.1	691.9	96.9	23.1
S3	1420.2	26.5	6.6	1168.3	189.7	44.3
S4	21.0	2.0	1.0	201.8	61.3	34.2
S5	1579.0	17.0	2.7	1272	166.8	28.3
S6	1300.5	39.1	5.3	1177.5	239.4	63
S7	213.9	2.5	0.6	517.5	59.1	13
S8	1445.0	32.3	1.1	1093.8	184.7	19.9
S9	46.9	4.0	1.7	309.3	88.7	31
S10	295.4	7.9	1.8	448.2	51.1	11.1
S11	8.7	0.9	1.2	47.8	11.5	4.4
Overall	638.5	12.7	2.1	661.5	109.6	26.6

pattern. For example, as the budget ratio increased from 0.5 to 2 (i.e., from S9 to S8), the number of cuts required by CUT2 increased by 108%, whereas the number of cuts required by CUT3 decreased by 36%. Finally, observe from the results for S10–S11 that as the proportion of period (σ) for which no competition exists increases, the solution effort of all three exact methods decreases as conjectured in Remark 5. For instance, the number of cuts required by CUT1 decreased from 448.2 to 47.8 as the value of σ increases from 0.3 to 0.7, while the CPU time decreased from 295.4 to 8.7 seconds. This result supports our conjecture that a dominant precompetition portion in the objective function will result in fewer cutting planes at optimality.

An interesting question is how large instances can become before our best exact method (CUT3) fails to converge in a reasonable amount of time. To address to this question, we conducted an additional experiment in which CUT3 is executed on different sets of test instances having various sizes. In particular, we further examined problem instances generated according to configurations similar to S1, whose instances were relatively easy to solve, and S3, whose instances were substantially more difficult to solve (Table 2). Recall that given $|N|$, S1 has $|M| = |N|/2$ and S2 has $|M| = \frac{3}{2}|N|$, while both sets are configured as $|O| = |N|/2$, $\rho = 0.5$, $\frac{B}{C} = 1$, and $\sigma = 0$. Let C1 and C3 denote problem configurations corresponding to S1 and S3, respectively. We generated eight additional test sets, each containing 10 instances, for each combination of $|N|$=16, 20, 24, and 28, and for configurations C1 and C3. Figure 2 displays the average CPU times consumed and cuts generated by CUT3. Because CUT3 was unable to produce solutions for C3 instances having $|N|$=24 or 28 within one hour of CPU time, we omit those results from the figure. Note that both CPU time and number of cuts reveal explosive growth patterns as $|N|$ increases. This behavior illustrates the need of quick and effective heuristic methods for large-scale problems.

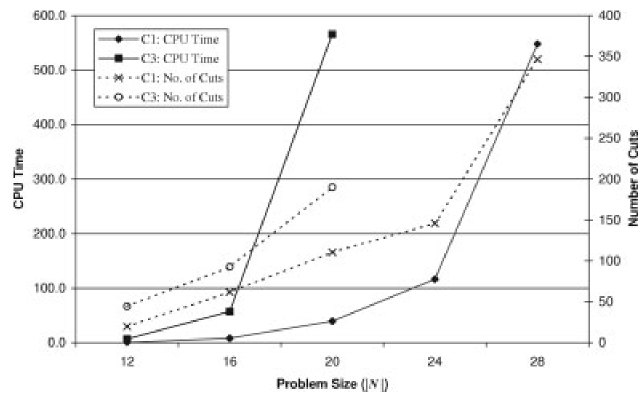

Figure 2. Computational effort trajectories of CUT3 as $|N|$ increases.

Accordingly, we examined the performance of three heuristic methods for solving test sets S1–S11. Recall that these heuristics function by relaxing the follower's problem to be a linear program, thus restricting the leader's problem. Therefore, the objective value obtained from the heuristic is a lower bound on the true objective function value corresponding to the heuristic solution when the leader's variables are restricted to be integer. To evaluate the quality of our heuristic solutions, we record the solution obtained from the heuristic and then solve the follower's (integer) problem to optimality. The objective value of the follower's (integer) problem in response to the heuristic solution minus the leader's budget is the value that we report as the heuristic objective value.

Letting $z^\star$ and z, respectively, denote the optimal objective value and the value that each heuristic method yields, we define the relative optimality gap as $[(z^\star - z)/z^\star]100\%$ and absolute optimality gap as $z^\star - z$. The average optimality gaps obtained by our heuristics are displayed in Table 3. All three methods terminated in 0.3 seconds for all S-set test instances. First, observe that RM exhibits the best performance by yielding the smallest average optimality gaps (an average of 3.1% relative and 220.5 absolute optimality gaps), while MRM and

Table 3. Average optimality gap produced by heuristic methods for S1–S11.

Set	Relative optimality gap (%)			Absolute optimality gap		
	RM	SRM	MRM	RM	SRM	MRM
S1	0	0	0	0	0	0
S2	0.4	3.7	6.3	41.5	352.2	644.1
S3	1.8	5.2	5.6	340.1	922.4	994.1
S4	1.6	7.5	7.5	198.9	767.8	767.8
S5	0.7	6.9	3.5	77.6	563.3	341.5
S6	21.1	29.6	30.0	251.8	653.8	632.9
S7	0.1	0.4	0.6	21.6	55.7	76.1
S8	3.6	11.5	6.0	545.5	1696.4	899.2
S9	0	1.1	1.1	0	73.7	73.7
S10	3.8	48.9	29.8	747.0	9619.4	5817.2
S11	0.6	43.2	27.7	202.0	14723.1	9510.9
Overall	3.1	14.4	10.7	220.5	2675.2	1796.1

Table 4. Average optimality gap produced by heuristic methods for L1–L11.

Set	Average relative solution quality			Average CPU time (sec)		
	RM	SRM	MRM	RM	SRM	MRM
L1	1.00	1.00	1.00	1.4	1.2	1.2
L2	1.00	0.97	0.97	14.9	3.4	6.7
L3	1.00	0.96	0.97	76.7	7.5	14.8
L4	1.00	0.95	0.97	3.7	0.9	2.7
L5	1.00	0.96	0.97	52.4	15.6	20.7
L6	1.00	0.80	0.80	5.7	3.4	5.8
L7	1.00	0.99	0.99	12.5	3.5	7.9
L8	1.00	0.87	0.96	36.0	3.7	15.5
L9	1.00	1.00	1.00	6.8	3.3	3.3
L10	1.00	0.49	0.74	111.7	0.6	2.7
L11	1.00	0.50	0.62	193.4	0.6	2.0
Overall	1.00	0.86	0.91	46.9	4.0	7.6

SRM produced the second- and the third-best performances. In particular, RM yielded average optimality gaps less than 1% for six problem sets (S1, S2, S5, S7, S9, and S11).

All three heuristic methods found an optimal solution for each S1 instance. Furthermore, RM found an optimal solution for each S9 instance. Observe that solution quality decreases as ρ decreases (see optimality gaps of S7, S2, and S6); this corresponds to the previous observation that the computational effort required to solve problem instances to optimality increases as ρ decreases. Based on these observations, we conclude that the problem becomes more difficult when ρ is small, because the follower can reduce the leader's revenue by a factor of $1 - \rho$. Hence, the leader's objective value can be significantly worse at a suboptimal solution than the optimal objective values. Also, note that all three heuristic methods display solid performances for S1, S7, and S9. This implies that the proposed methods are well applicable especially when the number of market segments is relatively small (S1), the leader has a better brand image (S7), or the size of the leader company is relatively small (S9).

Next, we examine the quality and computational effort of each heuristic applied to the larger L1–L11 instances. Table 4 displays CPU times and solution quality relative to RM, defined as $1 + (z_\bullet - z_{RM})/z_{RM}$ where $z_\bullet$ denotes the objective value produced by heuristic method.

For L1–L9 instances, in which $\sigma = 0$, RM consumed roughly 4.94 times the CPU time required by SRM, and roughly 2.67 times that required by MRM. Similar to the results for smaller test sets, all three methods display consistent performances for test sets L1, L7, and L9. SRM and MRM also achieved 5% or less relative objective gaps with respect to RM, with the exception of L6 instances, for which the relative gap was 20% in both methods, and L8 instances, for which SRM solutions revealed an average relative gap of 13%. However, observe that RM contrasts more with SRM and MRM when solving L10 and L11 instances: RM consumed significantly more CPU time than SRM or MRM, but produced far better quality solutions. In consequence, our recommendation is to use RM when the problem size permits the execution of RM. For larger instances, MRM can reasonably be used as an alternative. Nonetheless, when the value of ρ is small, or when σ is large enough to impact the optimal solution, there exists a risk that MRM will yield a poor quality solution.

6. SUMMARY AND FUTURE RESEARCH

In this article, we start with the premise that a firm's product introduction strategy often needs to be conducted with subsequent predatory behavior in mind. This problem can be modeled as a three-stage mixed-integer programming problem, in which stage one decisions are made by the leader, stage two decisions are made by the follower, and stage three decisions are made by the market segments. We demonstrate that this problem can be solved using a decomposition strategy based on an underlying bilinear programming structure. This formulation can then be improved by redefining the leader's decision variables, which lead to stronger cutting planes obtained from the decomposition algorithm. Our computational results demonstrate the efficacy of our reformulated decomposition model as compared to the original models, and also indicate that a mathematical programming-based heuristic tends to identify near-optimal solutions on our randomly generated test instances.

For future research, we believe that there exists a need for consistent heuristics to tackle the case in which there are many potential products to consider ($|N|$ is large), especially when the market share parameter (ρ) is small and/or when there is a significant amount of time in which the leader controls a monopoly (i.e., when σ is high). We anticipate that a metaheuristic approach that does not rely on mathematical programming or construction principles would be a promising avenue. Related studies might also analyze the case in which the follower acts in its own best interest, rather than in a predatory manner. Such a study would shed light on the potential profit advantages that could be realized when a firm leads rather than follows product development in an industry. Another fruitful line of investigation would be on consumer choice behavior: an explicit treatment of how individual consumer choices give rise to market segments and their preference lists would provide a rich set of special cases of our model in this article. It would also be of interest to expand this game so as to reflect the dynamic nature of real-life industry competition more faithfully. For instance, the competition may span more than two periods with firms taking turns in responding to its competitor's product introductions. Another example would be a surprise tactic that the leader can consider: Suppose that the leader introduces some of its products to elicit a certain response from the follower, and then withdraws those products from the market after the follower commits to its own product line. If the leader has enough resources to entertain this idea, it would be able to partially hide its ultimate strategy, making its survival more likely. (Naturally, this game would assume that the follower does not take this strategy of the leader into account, i.e., surprise only works if the other side cannot anticipate it.) Neither of these games are easy to analyze, as they appear to involve complex multi-level programming approaches. Our future research may seek to explore sophisticated exact or heuristic approaches capable of dealing with such games.

Finally, the research conducted in this article assumes symmetric knowledge of all data. It is possible in some scenarios that the follower will not have an identical perception of customer demands and market segment priorities, and will thus act in a manner that is optimally predatory in their estimation, but not necessarily in the leader's estimation. Hence, an interesting line of research would explore ways in which the leader can exploit this asymmetric knowledge to induce a less-damaging predatory action by the follower.

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